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Studying turbulence using direct numerical simulation: 1987 Center for Turbulence Research NASA Ames/Stanford Summer Programme

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This paper is an account of a summer programme for the study of the ideas and models of turbulent flows, using the results of direct numerical stimulations of the Navier–Stokes equations. These results had been obtained on the computers and stored as accessible databases at the Center for Turbulence Research (CTR) of NASA Ames Research Center and Stanford University. At this first summer programme, some 32 visiting researchers joined those at the CTR to test hypotheses and models in five aspects of turbulence research: turbulence decomposition, bifurcation and chaos; two-point closure (or k-space) modelling; structure of turbulent boundary layers; Reynolds-stress modelling; scalar transport and reacting flows.

A number of new results emerged including: computation of space and space–time correlations in isotropic turbulence can be related to each other and modelled in terms of the advection of small scales by large-scale motion; the wall layer in turbulent boundary layers is dominated by shear layers which protrude into the outer layers, and have long lifetimes; some aspects of the ejection mechanism for these layers can be described in terms of the two-dimensional finite-amplitude Navier–Stokes solutions; a self-similar form of the two-point, cross-correlation data of the turbulence in boundary layers (when normalized by the r.m.s. value at the furthest point from the wall) shows how both the blocking of eddies by the wall and straining by the mean shear control the lengthscales; the intercomponent transfer (pressure–strain) is highly localized in space, usually in regions of concentrated vorticity; conditioned pressure gradients are linear in the conditioning of velocity and independent of vorticity in homogeneous shear flow; some features of coherent structures in the boundary layer are similar to experimental measurements of structures in mixing-layers, jets and wakes.

The availability of comprehensive velocity and pressure data certainly helps the investigation of concepts and models. But a striking feature of the summer programme was the diversity of interpretation of the same computed velocity fields. There are few signs of any *convergence* in turbulence research! But with new computational facilities the divergent approaches can at least be related to each other.

1. Introduction

Recent developments in computing power have made it possible to compute accurately solutions to the Navier–Stokes equations for three-dimensional unsteady flows at Reynolds numbers of the order of 100 (based on a characteristic fluctuating velocity and turbulent lengthscale). So far, these computations of direct simulations can only be performed for flows bounded by surfaces with simple geometrical shapes. Even for simple geometries, each flow takes a substantial time to program and run on the larger computer systems. So it is not possible to ask a research centre specializing in computational fluid dynamics to compute instantly a new flow (within this Reynolds-number range). However, when the velocity and pressure fields of a certain number of flows have been computed and stored, they can readily be used to explore different ideas and theoretical models of these flows. An important application of this understanding is to improve approximate models of turbulent flows that are less expensive (in computer time) and also to reduce their inevitable limitations.

Since the results of these direct simulations are acceptably accurate solutions of exact equations, they have the same status as experiments and will be referred to as 'data'. A computationally derived data resource is analogous to the data sets of measurements that from time to time have been made available for researchers-at-large to examine, such as the well-known sets of meteorological, oceanographic and laboratory turbulence measurements. It is important to emphasize that most of these high-Reynolds-number flows cannot yet be computed exactly.

Data sets of computer-generated flow fields differ from experimental ones in the extra detail that is available; velocity, vorticity, pressure fields in time and in three dimensions. Some errors of experimental measurements are eliminated, such as the use of Taylors' hypothesis. (The results on fluctuating pressure are a particular feature of direct simulations, because they cannot yet be measured completely satisfactorily.) Furthermore, with present computing power and recent developments in software, it is possible for models and hypotheses to be tested readily within days or weeks, rather than months or years! But then the question arises as to which models are really suitable or worthwhile for this kind of testing and investigation.

Over the past decade, researchers at Stanford University and NASA Ames (at the Centre for Turbulence Research) have developed the techniques of direct numerical simulation to compute a range of flows chosen to emphasize many of the important mechanisms and phenomena of turbulence (Rogallo & Moin 1984). The development and application of these techniques now form the major part of the programme of the recently formed Centre for Turbulence Research, housed at Stanford University and at NASA Ames. The Centre's inaugural programme in 1987 was a new kind of workshop or summer school bringing together a group of about 32 invited researchers from outside with about 20 researchers at the Centre. The visitors were divided into five groups, whose titles are given as the titles of §§2–6 of this report:

Stochastic Decomposition, Bifurcation and Chaos.
Two-point Closure (k-space).
Structure of Turbulent Boundary Layers.
Reynolds-Stress Modelling.
Scalar Transport and Reacting Flows.

Each group aimed to explore certain projects, suggested by the visitors, using the computational facilities and the computer-generated database over a four-week period. (The visitors had been invited on the basis of suitability of their proposed projects.) The facilities available were much of the capacity of a CRAY XMP and a CRAY II and several advanced (IRIS) workstations.

As part of the programme, 8 review tutorials were given on particular aspects of turbulence research: Graphical Display of Data (B. Hesselink), Rapid Distortion

Theory (J. C. R. Hunt), Phase-Plane Methods (B. Cantwell), Conditional Sampling and Stochastic Estimation (R. J. Adrian), Probability Density Function Modelling (W. Kollmann), EDQNM Turbulence Theory (J. P. Bertoglio), Dynamical System Theory (L. Keefe). Also there was a short colloquium on research into coherent structures with contributions by S. J. Kline, P. Bradshaw, R. J. Adrian, P. Moin, J. C. R. Hunt, J. Kim, M. T. Landahl, A. K. M. F. Hussain, J. Jimenez.

Short research seminars were also given during the programme. But the most notable feature of the organization was the requirement placed on participants to present their plan of work after 2 days, their interim results after 2 weeks, and their main results on the last day of the programme. This was a demanding schedule, but was probably necessary to accomplish what was achieved.

As a result of the summer programme, a number of new results emerged and, perhaps as important, new connections have developed between different schools of turbulence research. In future, this kind of summer programme may well become an important part of research in fluid mechanics and therefore we hope that this report will be of interest to readers of the *Journal of Fluid Mechanics.*

A detailed proceedings of the summer programme will be produced by the Center for Turbulence Research early in 1988. This report was largely compiled from the summary statements in the draft proceedings by P. Moin, J. Kim, N. N. Mansour, R. S. Rogallo and W. C. Reynolds (the organisers of the summer school).

2. Stochastic decomposition, bifurcation and chaos

This group consisted of four loosely interrelated projects with the common objective of understanding the mechanics of wall-bounded turbulent flows. All projects used the database of the full velocity and pressure field for the flow in a turbulent channel at Reynolds number of about 3300 (based on the mean flow) (Kim, Moin & Moser 1987) or the two-point velocity correlation tensor computed from it (Moin & Moser 1988).

The invited participants were:

R. J. Adrian (University of Illinois).
N. Aubry (Cornell University).
J. C. R. Hunt (University of Cambridge).
J. Jimenez (Universidad Politecnica, Madrid, and UAM-IBM, Spain).

The local participants were:

L. Keefe (CTR).
P. Moin (Stanford University and NASA Ames).
R. D. Moser (NASA Ames).

The analysis of shear-free boundary layers by *Hunt*† (1984) shows that near a rigid surface the velocity correlations are self-similar, i.e. the two-point correlation of normal velocity v, when normalized by the intensity $\overline{v^2}$ at the point further from the wall, is of the form $f(y/y_1)$. Using numerically generated correlations, this hypothesis was applied to turbulent boundary layers and channel flows and was shown to be valid except near the channel centreline or layer edge. A similar collapse was obtained for R_{uv} in the log layer. Hunt, Kaimal & Gaynor (1988) had already shown that f is linear in y/y_1 for convective shear-free atmospheric boundary layers.

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† The names in italics are those whose particular research in the relevant group is being described.

Comparison with these results clearly shows that shear reduces the correlation length of the normal velocity, in the normal direction. Using the same normalization, the variation of eddy scales in the spanwise direction was also investigated, and a strong dependence on shear was found. These results should be very useful in turbulence modelling and in other applications where two-point correlation data are used (e.g. see below).

The eigenfunctions of the spatial two-point correlation tensor were used in *Aubry's* dynamical-systems representation of wall-layer turbulence. In their previous work Aubry *et al.* (1987) used eigenfunctions obtained from the experimental measurements of Herzog (1986) in the near-wall region of a turbulent boundary layer. Employing these eigenfunctions, which appeared physically as roll-cells, they obtained a highly truncated solution of the Navier–Stokes equations and used methods of dynamical-system theory to analyse the results. The results exhibited intermittency which was associated with the bursting events in the sublayer. A similar analysis was performed at the CTR using the eigenfunctions computed from simulation databases (P. Moin & R. D. Moser 1987, in preparation). The results were different from those of Aubry *et al.* In particular, limit-cycle behaviour was observed just prior to intermittency, rather than the fixed-point behaviour found previously. As a result the character of the intermittency is different and it is significantly more sensitive to the bifurcation or eddy viscosity parameter. In view of the significance of this work in relating dynamical-systems theory to the structure of turbulence, further work is required to determine the sensitivity of the results to various computational parameters and other inputs.

Two-point correlation data were also used to extend, to three dimensions, the work of *Moin, Adrian* & Kim (1987) on stochastic estimation of 'conditional eddies', i.e. eddies characterized by the velocity at one or two points meeting particular conditions (e.g. $u > 0$, $v < 0$). Previous work had applied only to motions in planes transverse to channel flow. With stochastic estimation, one approximates the average structure of an eddy, in terms of the conditional velocity at a point and the two-point correlation tensor. In addition, the theory was extended to include specification of conditions at more than one point. An important result of this investigation was the verification that linear stochastic estimation (in which the eddy velocity is everywhere proportional to the velocity at one or more points in the eddy) indeed provides an accurate representation of conditional eddies.

It was also shown that two-point stochastic estimates of the conditional eddies provide reasonable representations of the instantaneous flow structures. This technique is capable of generating the asymmetric structures that occur in the instantaneous flow field. Using conditions obtained from shear layers in the instantaneous field (see below), a simplified model of the shear layers was proposed which consisted of inclined vortical structures surrounding each shear layer.

Perhaps the most dramatic observation in this group was the discovery that turbulent channel flow contains a high density of strong, and highly visible, shear layers. The shear layers are regions of strong spanwise vorticity protruding from the wall region into the outer layers. Apparently, the dominance of these shear layers, at least for the low Reynolds numbers considered here, has been overlooked previously. More importantly, the patterns of these shear layers, depicted in contour plots of spanwise vorticity in planes normal to the wall and in the flow direction, strongly resembled those in *Jimenez's* (1987) two-dimensional numerical solutions of the equations near *transition* with finite-amplitude disturbances. Although in channel flow these shear layers are three-dimensional, the generation mechanism appears to be the same as in the two-dimensional case. The shear layers were followed in time and this generation process was observed directly. Based on these observations a simple model was proposed to explain vorticity ejection from the sublayer and the production of the shear layers. This model is essentially equivalent to the mechanism responsible for the instability of two-dimensional Tollmien–Schlichting waves. Finally, by reducing the size of the computational box, futile attempts were made to study the dynamics of one shear layer in isolation (in the absence of complex interactions with other structures). One of the by-products of this latter study was an interesting numerical solution which displayed three-dimensional turbulence on one side of the channel, and essentially two-dimensional flow on the other. The average wall shear stress of the turbulent layer falls between the values characteristic of the two-dimensional nonlinear solutions and the three-dimensional turbulent solutions.

3. Two-point closure (k-space)

The projects of the k-space group consisted of six independent lines of enquiry with a common theme of the interaction among scales of motion in turbulence. The studies were almost entirely limited to homogeneous turbulence.

The invited participants were:

J. P. Bertoglio (Ecole Centrale de Lyon).
J. C. R. Hunt (University of Cambridge).
P. Orlandi (University of Rome).
R. Schiestel (Institut de Mécanique de Statistique de la Turbulence, Marseille).
A. Yoshizawa (University of Tokyo).

The local participants were:

J. C. Buell (NASA Ames).
G. Coleman (Stanford University).
J. H. Ferziger (Stanford University).
R. S. Rogallo (NASA Ames).
A. A. Wray (NASA Ames).

Bertoglio wished to test the accuracy of the assumptions within the EDQNM† (Lesieur 1987) closure at a deeper level than has been previously done. An important step in this closure is the estimation of a Lagrangian timescale for the velocity of two particles at an initial spacing k^{-1}. This is assumed to be a functional of $E(k)$. Bertoglio judged several candidate functionals by the degree to which they 'collapse' the two-time velocity autocorrelations in direct simulations of isotropic turbulence. This approach appears very promising and good collapse has been achieved at the higher wave-numbers. This work is the continuation of a joint effort between Lyon and Stanford (Lee *et al.* 1987).

The EDQNM theory is purely statistical in nature, whereas many important turbulence problems are dominated by the presence of persistent coherent structures. Bertoglio's second goal was to determine how well EDQNM copes with such structures. An experimental programme has been started at Lyon using propellers to inject known coherent structures into a turbulent flow, and to conduct a parallel numerical simulation (Michard *et al.* 1987). Comparisons between experiment, simulation, and theory for this flow should illuminate any difficulties that the

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† Eddy-damped quasi-normal Markovian.

statistical theory has with imbedded coherency. During the workshop Bertoglio attempted to define initial conditions for such a simulation. The energy in the simulation peaked at the blade-passing frequency of the propellers while that in the experiment peaked at their rotation frequency. It should be possible to solve that mystery, but there was not enough time during the workshop to do so.

Orlandi's project concerned the use of EDQNM as a subgrid (or supergrid) model for a large-eddy simulation. At a minimum, such models must accurately account for the energy transfer between the (computationally) resolved and unresolved scales. In the EDQNM theory, the net transfer into wavenumber k is calculated as the integral over interacting triads (k, p, q) of a functional of $E(k)$, $E(p)$, and $E(q)$. Orlandi compared the transfer spectra predicted by EDQNM with that measured in a direct simulation of isotropic turbulence for both full and truncated energy spectra. The ability of EDQNH to predict supergrid transfer, as well as subgrid transfer, was confirmed. The utility of EDQNM as a subgrid model has been previously demonstrated by Chollet (1982). Then, if the spectrum of the unresolved scales can be estimated, EDQNM can account for their contribution to the energy transfer. In the simulation however, the effect of the unresolved scales must appear as additional terms in the *momentum* equations. The subgrid term is usually modelled by a gradient diffusion form with an eddy viscosity determined from the subgrid transfer (e.g. Kraichnan 1976; Lesieur 1987). The *supergrid* term is presumably some sort of forcing but a gradient diffusion form (involving energy with a negative eddy viscosity) does not seem physically correct. Another possibility is the application of a mean strain that is uniform in space but random in time. The art of driving simulations at the large scales, as was done by Hunt *et al.* below, is currently not well understood (see also Yeung & Pope 1987).

While the EDQNM model appears to be reasonably accurate and tractable in isotropic turbulence, in anisotropic flows it is far less tractable, requires additional assumptions, and is much more expensive to compute. Because of this, the theory has only received limited attention for anisotropic flows (Cambon, Jeandel & Mathieu 1981). These are however very important in the context of subgrid models for LES calculations because as the grid resolution increases, the subgrid contribution approaches homogeneity much more rapidly than it approaches isotropy. *Coleman* and *Ferziger* considered the possibility of a Galerkin approach to simplify the EDQNM calculation. The angular distribution of velocity correlations over spherical shells (which are, in the proper variables, uniform in isotropic flow) would be represented by the weighted sum of a small number of smooth basis functions. Coleman and Ferziger attempted to estimate the number of basis functions required by inspecting the angular distribution of the Reynolds-stress-spectrum tensor in a direct simulation of homogeneous sheared turbulence. The distributions were quite smooth and the authors speculated that they could be represented by a sum of two or three functions. There appear to be several important issues that were not covered in a general way: the choice of the coordinate system and the choice of the dependent variables. In any expansion technique the choice of variables, both dependent and independent, is crucial. In this case some clues might be extracted from rapid-distortion theory, from consideration of the principal axes of the stress and mean strain-rate tensors, and from the manner in which the mean flow gradient enters the EDQNM equations.

The project of *Hunt, Buell,* and *Wray* concerned the relation between space and time spectra (or correlations), and their dependence on the reference frame (Eulerian or Lagrangian). In particular, they wanted to determine how the advection of small weak scales by large strong ones influences the Eulerian time spectra at high frequency and wavenumber. The results support the assertion of Tennekes (1975) that such advection dominates the frequency spectrum at high frequency, but indicates that Hunt's earlier proposal relating the time–space spectrum to the space spectrum was oversimplified (Carruthers & Hunt 1986). During the project, following comments by Kraichnan, Hunt reworked the analysis using a more realistic p.d.f. for the advecting velocity and derived a simple relationship (originally proposed by Chase 1969) between the space–time and spatial spectra which agrees with the simulation and, in one dimension, with an earlier proposal of Wills (1971) for the wavenumber frequency spectrum of pressure fluctuations. Some anomalies were observed in the computed data; these appear to be a consequence of the small statistical sample of forced modes, the small range of spatial scales that could be retained, and the low Reynolds number that was required for adequate numerical resolution.

Schiestel's objective was to test a model (Schiestel 1987), in which the equation for the Reynolds-stress-spectrum tensor is integrated, in wavenumber space, over spherical shells rather than over the entire space as done in classical one-point Reynolds-stress formulations. As a result, some scale information is retained. But in its application this method is closer in spirit to one-point closures than to two-point closures. Within each shell, one must model the same quantities as in classical one-point closures (pressure–strain, dissipation, etc.) and in addition, model transfers between shells. These transfers are globably conservative and do not appear in the one-point approaches. Shiestel's goal during the summer programme was to compare the models for these terms with data from direct simulations. He hoped to determine the accuracy of the models he is currently using, and to get some clues to aid in their improvement. As one would expect, some of the terms were modelled rather well while others were not, and, unfortunately, there was not enough time to consider improvements. The statistics taken from the simulations were rather noisy at the larger scales because of the small sample and sometimes biased at the small scales owing to mesh anisotropy. However, the use of numerical simulation data is the only way to test this aspect of the theory.

Over the past several years, *Yoshizawa* has worked out a formal two-scale expansion of the Navier–Stokes equations in which the interaction between the scales becomes explicit (Yoshizawa 1985). The scales are disparate in both space and time, and are separated by formally averaging over an intermediate scale, at which Taylor series expansion of the large scales is assumed to be valid and at which averages of the small scales are assumed to be statistically converged. The interaction terms in general depend upon deterministic features of the large-scale field (its derivatives) and statistical features of the small-scale field (local correlations). The disparity of the spatial scales leads, at small scale, to homogeneous turbulence at lowest order, and the timescale disparity leads to its isotropy. The required statistics of the small scales are in turn modelled by the DIA formalism. At higher order the small scales become anisotropic. Within this framework (TSDIA) it is possible to find the form (formally, the asymptotic expansion) of terms that must be modelled in one-point closures, for example the diffusion of kinetic energy. A model is then postulated by replacing the gauge functions in the expansion with 'model constants'.

It was Yoshizawa's hope to be able to test several of these models and to estimate the contribution of the higher-order terms, using simulation data to determine the constants. Unfortunately some model terms could not be computed because the required statistics could not be extracted from the database in the time available. In addition, the data that were available were not really adequate for the simultaneous determination of several constants. The sample was too small, as was the Reynolds number. Yoshizawa was then forced to omit most of the new terms suggested by TSDIA. When this was done the models fit the data well (with one notable exception) with constants close to the values previously predicted. For example, the fact that rotation reduces the dissipation rate was correctly predicted. The exception was the case of turbulent diffusion in homogeneous shear of a passive scalar having a mean gradient in the stream direction. In this case the mean gradient itself changes with time but was in fact held fixed in the simulation used by Yoshizawa. A later simulation treated the case of changing mean gradient but it is not clear which case is appropriate for testing Yoshizawa's model.

4. Structure of turbulent boundary layers

Unlike the other groups, the people in the Turbulence Structure group were primarily experimentalists, with the exception of Landahl. The summer programme thus provided a unique opportunity for these experimentalists to assess the numerical data in comparison with their own experimental data, and to extend their previous work using full three-dimensional turbulence fields. The group expressed a particular interest in investigating temporal evolutions in addition to the spatial variations of the organized turbulence structures.

The invited participants were:

H. Alfredsson (KTH, Stockholm).
A. Johansson (KTH, Stockholm).
R. Blackwelder (University of Southern California).
J. Swearingen (University of Southern California).
Y. Guezennec (Ohio State University).
D. Henningson (FFA, Sweden).
A. K. M. F. Hussain (University of Houston).
J. Joeng (University of Houston).
M. T. Landahl (MIT and KTH, Stockholm).
M. K. Breuer (MIT and KTH, Stockholm).

The local participants were:

J. Kim (NASA Ames).
P. R. Spalart (NASA Ames).
U. Piomelli (Stanford University).
S. K. Robinson (NASA Ames).

During the first two days of the summer programme, the following items were identified as unifying themes for the group: detect significant structures; compare with experimental results; observe space–time evolutions; implement improved averaging schemes; investigate flow instability.

With these unifying themes in mind, the group was divided into several teams and each team proceeded to investigate the organized structures in wall-bounded shear flows using the databases generated by Kim *et al.* (1987) (channel) and by Spalart (1986) (boundary layer). Data were prepared in time intervals short enough ($\Delta t^+ = 3$) to accommodate the study of the temporal evolution. ('Wall units' are used here based on the length ν/u_* and time ν/u_*^2, where u_* is the wall friction velocity).

The most significant result from the Turbulence Structure group as a whole concerned the temporal evolution of the organized structures in wall-bounded shear flows. Several different detection schemes, ranging from a simple visual method to rather sophisticated iteration procedures, were used to detect the organized structures. The resulting structures were slightly different from each other, since each scheme emphasized different aspects of the structures; however, these structures were also related to each other in many respects. For example, the internal shear layer investigated by Alfredsson and Johansson was generally observed between the structures associated with fourth- and second-quadrant events (i.e. $u > 0$, $v < 0$ and $u < 0$, $v > 0$) investigated by Guezennec. In all cases, the structures retained their coherence for a much longer time than expected, and consequently they could be tracked over a long streamwise extent. Typically, the organized structures persisted over a period on the order of $t^+ \approx 100$, and they could be tracked over a distance of the order of $x^+ \approx 1000$. The pictures emerging from these investigations suggest that the organized structures do not go through violent break-up processes, as suggested by previous studies, but rather diffuse slowly into incoherent motions. The spatial structure, however, was highly localized in space with small-scale motions within the structure. When such a structure passes a fixed probe in space, it can leave signatures that might look like a violent break-up process. To confirm this conjecture, it would be worthwhile in a future study to perform an *in situ* comparison between the spatial distribution of a detected structure and the temporal signature at a fixed point when the structure passes by.

Brief summaries of the results of particular projects are given below.

Alfredsson and *Johansson* (in collaboration with *Kim*) studied the formation and evolution of shear-layer-like flow structures in the buffer region of wall-bounded turbulent shear flow that were associated with turbulence production. The structures were found to retain their coherence over streamwise distances on the order of 1000 viscous length units, and propagated with a constant velocity of about 10.5 u_* throughout the near-wall region. The shear-layer structures were found to be important contributors to the turbulence production of turbulent energy: the conditionally averaged production at the centre of the structure was almost twice as large as the long-time mean value. Individual shear layers often showed a strong spanwise asymmetry which was lost in conventional conditional-averaging procedures.

Breuer and *Landahl* (in collaboration with *Spalart*) performed numerical simulations in which structures similar to those described by Alfredsson and Johansson were used as initial velocity fields, surrounded by a laminar boundary layer. The objective of this study was to investigate the dynamics of such structures in isolation, which made them easier to detect and follow in time. It was found that the structure associated with a fourth-quadrant event upstream of a second-quadrant event grew much more rapidly than that associated with a second-quadrant event upstream of a fourth-quadrant event. This is consistent with the fact that one finds more energetic events of the former type than of the latter in turbulent flows.

Guezennec (in collaboration with *Piomelli* and *Kim*) implemented several ensemble-averaging techniques (VITA (Blackwelder & Kaplan 1976), quadrant technique, techniques based on wall shear, etc.) to determine organized structures in the wall-bounded flows. The results were in good agreement with his experimental results. It was found that the size of the detected structures in wall units was a function of Reynolds number, and approximately scaled with the boundary-layer thickness. The ensemble-averaging process was improved by taking the asymmetry of the turbulence structures into consideration. The resulting structures were strongly asymmetric, suggesting that conventional ensemble-averaging schemes are misleading in that respect. It was also observed that these structures were persistent over a time on the order of 50 viscous time units.

Hussain and *Jeong* (in collaboration with *Kim*) applied a conditional-sampling technique designed to detect coherent vorticity through an iteration procedure to the above-mentioned databases, as well as to a homogeneous shear flow field, to educe coherent structures from each flow. Many characteristics of the detected structures were quite similar to those of the mixing layer observed experimentally by Hussain and his colleagues (Hussain 1986): the topology consisted of saddles and centres, the saddle regions being the locations of maximum incoherent Reynolds shear stress and maximum shear production. From contours of coherent and incoherent tubulent energy and pressure, the relative amounts of normal and shear production, and the intercomponent kinetic energy transfer by pressure fluctuations were deduced. The effects of shear and the wall on the coherent structure was investigated by comparing the structures in the wall region (high shear) with those in the outer layer (low shear), and those of the *homogeneous* shear flow simulated by Rogers *et al.* (1986).

Hussain (in collaboration with *Kim* and *Spalart*) also studied the propagation speeds of the velocity, pressure and vorticity by examining cross-correlations between two fields at different times (Hussain & Clark 1981). It was found that the propagation speeds for velocity and vorticity agreed with each other throughout the channel and boundary layer; at each height they were slightly less than the mean velocity, except in the wall region ($y^+ < 20$) where the propagation speed of $0.55U_c$ is very close to the value obtained visually by Alfredsson and Johansson. The propagation speed for the pressure was also constant in the wall region but much higher than that for velocity and vorticity ($0.75U_c$), whereas in the outer layer it was almost the same as them. An important result is that above the wall region Taylor's hypothesis for relating temporal and spatial changes of turbulence works well, provided the correct propagation speed is chosen; in fact better than theoretical estimates would indicate (e.g. Lumley 1965).

Swearingen and *Blackwelder* (in collaboration with *Spalart* and *Robinson*) investigated flow instabilities associated with shear layers by examining the structure of the normal shear layer ($\partial u/\partial y$) and the spanwise shear layer ($\partial u/\partial z$). They found that a strong shear and an inflexional velocity profile existed surrounding the low-speed region and, more importantly, these persisted up to 60 ν/u_τ^2 indicating sufficient time for an instability to develop. The low-speed streaks developed an oscillatory motion which increased in time (also indicative of instability) and eventually the undulating portion of the streaks appeared to break up into chaotic motion.

Landahl (in collaboration with *Kim* and *Spalart*) examined the basic hypothesis of his 'active-layer' model for wall-bounded turbulence (Landahl 1975). The model assumes that the nonlinear (fluctuation–fluctuation) terms are large only in a thin layer near the wall, and hence the turbulence in the region outside the active inner layer can be modelled as a linear response driven by the active layer. Preliminary investigation indicated that the nonlinear effects were indeed strongest near the wall with a maximum around $y^+ = 20$ and, outside the near-wall region, they involved primarily the cascading mechanism to dissipative scales of motion. This model may lead to a reasonably simple procedure for determining the Reynolds stresses and other statistical quantities through a comparatively simple linear calculation making use of a universal model for the nonlinear processes in the near-wall region.

Henningson and *Landahl* (in collaboration with *Kim*) used a kinematic wave theory to investigate the cause of the rapid growth of waves observed at the 'wingtip' of a turbulent spot in plane Poiseuille flow. It was found that the qualitative behaviour of the wave motions was well described by Landahl's (1972) breakdown criterion, which appears to control the wave selection procedure.

5. Reynolds-stress modelling

It is well recognized that full turbulence simulations will be limited for the foreseeable future to simple fundamental flows. In order to compute flows of engineering interest, turbulence models will have to be used in formulating the governing equations. These models are a 'pacing' item in the development of a computational fluid dynamics in practice. Traditional model development relies on formulating a closure model, whose prediction of, say, Reynolds stress, is then compared with experiment to assess its validity indirectly. No direct measurements are available (or are currently possible) for certain terms in the models, such as the pressure–strain term in second-order models, and therefore no direct assessment of the model is possible. However, direct simulation data can be used to compute the terms that need to be modelled and can be compared directly with the closure formulae to test their validity. This process should lead to a more systematic way of testing models. The objectives of the Turbulent Modelling Group were to develop and test closure models on the data set for boundary layers and homogeneous shear flows.

The invited participants in the modelling group were:

P. Bradshaw (Imperial College London).
J. G. Brasseur (Clemson University, South Carolina).
C. G. Speziale (ICASE; NASA Langley, Virginia).
J. Weinstock (NOAA/ERL/Aeronomy Laboratory, Boulder).
M. Wolfshtein (Technion, Israel).
J. C. R. Hunt (University of Cambridge).
D. Vandromme (CORIA-CNRS, Rouen).
H. Ha Minh (IMF-CNRS, Toulouse).

The local participants were:

J. Y. Chen (Sandia National Laboratories).
M. J. Lee (NASA Ames).
S. K. Lele (NASA Ames).
N. N. Mansour (NASA Ames).
U. Piomelli (Stanford University).
M. W. Rubesin (NASA Ames).
K. Shariff (NASA Ames).
T.-H. Shih (Center for Turbulence Research).
J. R. Viegas (NASA Ames).

As expected, most of the work was devoted to the assessment of existing turbulence models. At NASA Ames the model of Launder, Reece & Rodi (1975) had already been tested against the computed channel flow data (Mansour, Kim & Moin 1988), but the summer school provided an opportunity for various other modellers to test their models against full turbulence simulation data.

Weinstock and *Shariff* evaluated the theoretical predictions of Weinstock (1981, 1982), and Weinstock & Burk (1985) for the nonlinear component of the pressure–strain correlation – the return to isotropy or 'slow' term. They found that many of the features of the 'slow' term are reproduced by the theory. In particular the data indicate that the 'Rotta' coefficient, which multiplies the velocity anisotropy tensor, varies between components, and changes significantly when the turbulence kinetic energy 'changes' rapidly along the flow.

In a related study, *Rubesin*, *Viegas*, *Vandromme* and *Ha Minh* found that Reynolds-stress models in which the terms are assumed to be *linear* in the anisotropy tensor perform poorly for turbulent channel flows at different Reynolds numbers. For the homogeneous shear flow, it was found that pressure–strain models that are nonlinear in this tensor perform better. So it may be hoped that the same would apply to inhomogeneous shear layers such as boundary layers or channel flows.

Calculations of two-point correlations using rapid-distortion theory of turbulent flows in shear and near boundaries have suggested that the integral scale L_ϵ which controls dissipation is approximately controlled by the distance from the wall y and the shear scale $(v'/\mathrm{d}U/\mathrm{d}y)$. L_ϵ is the harmonic mean of the scales, multiplied by two given coefficients (Hunt, Stretch & Britter 1986). This formula was found by *Hunt*, *Spalart* and *Mansour* to describe L_ϵ for a wide range of wall-bounded turbulent shear flows, including reversing flows, except within about a distance L_ϵ from regions where $\mathrm{d}U/\mathrm{d}y = 0$.

Shih, *Mansour* and *Chen* evaluated nonlinear models for the return to isotropy and the rapid pressure-strain terms (Shih & Lumley 1985). In general, the models of the pressure–strain term performed well for the case of homogeneous turbulence under axisymmetric contraction, but were less satisfactory for the cases of homogeneous turbulence under plain strain. Shih, Mansour and Chen also compared the Reynolds-stress models with the direct simulation of homogeneous sheared turbulence computed by Rogers, Moin & Reynolds (1986).

Bradshaw, *Mansour* and *Piomelli* examined certain approximations to the 'rapid' or linear pressure–strain term in particular. In deriving the form of this term, they considered using the local mean velocity gradient $\partial U/\partial y$ outside the integral solution of the Poisson equation for pressure. It was found that away from the wall this approximation works well, in conformity with the use of the local $\partial U/\partial y$ in most current models, but close to the wall it failed. This failure is attributed to the fact that close to the wall the velocity gradient varies on a scale comparable with the correlation length of the fluctuating gradient. (This test was made with the channel flow data.)

A detailed study of a homogeneous shear flow field was carried out by *Brasseur* and *Lee*, where the intercomponent energy transfer by pressure–strain rate was investigated. It was found that the 'rapid' and 'slow' parts of the turbulent pressure were uncorrelated, providing strong justification for current modelling procedures. In addition, instantaneous events of high-transfer regions were studied in detail. These events were found to be highly localized in space and are imbedded in regions of concentrated vorticity.

In order to gain insight into the effects of rotation on the dissipation rate, *Speziale*, *Mansour* and *Rogallo* carried out a direct numerical simulation of decaying isotropic turbulence in a rapidly rotating frame. It was found that the primary effect of rotation is to shut off the energy transfer so that the turbulence dissipation is substantially reduced (see, for example, Bardina, Ferziger & Rogallo 1985). It was found that the anisotropy tensor remained essentially unchanged while the energy spectrum underwent a pure viscous decay. Rapid-distortion theory analysis reveals that the rate of change of the vorticity field is of the order of the product of the rotation rate and the vorticity, so that no Taylor–Proudman reorganization of the flow to a two-dimensional state was observed. Suggestions were made towards including the effects of rotation on the dissipation rate.

6. Scalar transport and reacting flows

This group conducted five projects aimed at developing improved understanding and modelling capabilities for turbulent flows with scalar transport or chemical reaction. Two projects used existing databases selected from previous simulations of incompressible homogeneous flows, both with and without mean deformation; two modified the basic program for homogeneous incompressible turbulence to account for simple chemical reactions; and one modified the code to include a Boussinesq buoyancy term.

The invited participants were:

W. T. Ashurst (Sandia Labs, Livermore).
S. El-Tahry (General Motors Research Laboratories).
J. C. Hill (Iowa State University).
M. J. Jennings (Illinois Institute of Technology).
A. D. Leonard (Iowa State University).
W. Kollman (University of California, Davis).
T. Morel (Integral Technologies, Chicago).
M. Mortazavi (University of California, Davis).

The local participants were:

S. Abrahamson (Stanford University).
J. H. Ferziger (Stanford University).
W. C. Reynolds (Stanford University and NASA Ames).
R. S. Rogallo (NASA Ames).
M. M. Rogers (NASA Ames).
C. J. Rutland (Stanford University).
K. Squires (Stanford University).

Using the database for homogeneous shear, *Kollmann*, *Mortazavi*, *Squires* and *Rogers* examined one-point probability density functions for the pressure gradient, conditioned on components of the fluctuating velocity and vorticity. The objective was to develop a firm basis for modelling quantities needed in p.d.f. treatments of turbulent flows (Kollmann & Janicke 1985).

They found that the expectation value of the fluctuating pressure gradient, conditioned on a turbulence velocity component, is linear in the velocity component over the range of velocity where an adequate sample existed. In contrast, the expectation values of pressure gradient conditioned with vorticity were found to be very small, indicating that the pressure gradient is statistically independent of the vorticity. It thus appears that the conditioned pressure-gradient term in homogeneous shear flow can be modelled as

$$\langle (1/\rho)\,\partial p/\partial x_i \,|\, u_1, u_2, u_3, \omega_1, \omega_2, \omega_3 \rangle = \alpha_{1i} u_1 + \alpha_{2i} u_2,$$

where $\alpha_1 (< 0)$ and $\alpha_2 (> 0)$ are constant. This model is likely to be useful in p.d.f. modelling of turbulence.

Morel, Jennings, and *Abrahamson* explored Morel's ideas for non-local modelling of turbulent scalar transport using integrals formed from two-point correlation functions. For homogeneous turbulent shear flow they examined the quantity

$$I_{ij}(t) = \int Q_{kk}(\boldsymbol{r},t)\, r_i r_j \,\mathrm{d}^3\boldsymbol{r},$$

where

$$Q_{ij}(\boldsymbol{r},t) = \overline{u_i(\boldsymbol{x},t)\, u_j(\boldsymbol{x}+\boldsymbol{r},t)}.$$

Morel's conjecture was that the component of turbulent transport of a scalar ϕ in the i^{th} direction might be modelled as $T_{\phi_i} = C\Phi_{,j} I_{ij}$ where Φ is the mean scalar field. This is of the form $T_{\phi_i} = -D_{ij}\Phi_{,j}$ which Rogers *et al.* (1986) found fit their direct simulations of homogeneous turbulent shear flow. However, Morel's model gives a symmetric D_{ij}, whereas Rogers *et al.* found that D_{ij} is non-symmetric. Thus, while Morel's model predicted the correct trends for the diagonal terms, it did not predict the off-diagonal terms.

The Rogallo (1981) code for homogeneous turbulence was modified by *Ashurst* to include a Boussinesq forcing term in the momentum equation. The objective was to study the effects of buoyancy on the relationships between vorticity and strain rate in homogeneous shear flow.

An unstable temperature gradient was imposed in the gravitational direction and the turbulence development was studied and compared to a zero-gravity isotropic decaying case. A computational box that was twice as long in the gravitational direction as in the horizontal was employed to permit the simulation to be run longer before the flow structures were affected by the imposed periodic boundary conditions. The simulations were begun from developed initial isotropic fields at $Re_\lambda = 7.8$ and run on a $64 \times 32 \times 32$ grid. The Rayleigh number based on the Taylor microscale varied from 21 to 93, and the Prandtl number was 0.7.

As expected, the Boussinesq term eventually caused the velocity fluctuations and length-scales in the gravity direction to grow. The relationship between the vorticity and the eigenvectors of the strain-rate tensor in the buoyant fields was qualitatively similar to that observed in incompressible homogeneous shear flow and isotropic decaying turbulence studied previously (Ashurst *et al.* 1987). Specifically, the vorticity has a large probability of aligning with the intermediate strain-rate eigenvector direction and at large strains this intermediate strain rate is extensional. This direction has a low probability of being aligned with the gravitational direction. The temperature field also behaves like the passive-scalar fields in incompressible shear flow in that the local temperature gradient is most likely to be aligned with the most compressive eigenvector of the strain-rate tensor. In addition, the scalar dissipation (conditioned on the kinetic energy dissipation) resembles the behaviour in incompressible shear flow, exhibiting a near linear dependence at large values. These results should be useful to developers of models for buoyant turbulent convection.

El-Tahry, *Rutland*, *Ferziger*, and *Rogers* modified the Rogallo (1981) code for homogeneous turbulence to study premixed turbulent flames in decaying isotropic incompressible turbulence. The objective was to determine the effect of turbulence on flame structure and speed over a range of Damkohler number. The results, though preliminary, are likely to be of interest to combustion modellers.

They considered the reaction $\mathrm{A} \rightarrow \mathrm{B}$, where A denotes premixed reactants and B denotes products. A temperature-dependent reaction rate was used, with the Arrhenius parameters adjusted to yield a flame width of about ten gridpoints (to resolve the structure) and a 15 °C temperature rise across the flame (to allow treatment as incompressible). In order to keep the flame thin compared with the turbulence scales and still resolve the flame structure, the Reynolds number had to be kept small, and the Taylor-microscale Reynolds number was only about 5 in their simulations. The Damkohler number was approximately 1.5. A $128 \times 128 \times 128$ grid was employed, with a uniform initial chemical distribution (premixed reactants) and a Gaussian temperature distribution sufficient to ignite the reaction. A purely laminar case was run for reference purposes.

The resulting flame front exhibited the characteristics of a wrinkled laminar flame; the local flame structure was everywhere similar to that of a laminar flame, with isotherms approximately parallel at about the same spacing as in the laminar case. The wrinkling of the instantaneous flame front appeared well correlated with the local velocity fluctuations and resulted in a mean flame thickness of about twice the local (laminar) flame thickness. The increased reaction area led to turbulent flame speeds about 20% larger than the laminar flame speed. As the flame propagated through the decaying hydrodynamic field it slowed, owing to the decaying turbulence intensity (decreasing roughly linearly with the r.m.s. turbulence intensity) and thickened slightly, perhaps owing to the increasing turbulent lengthscales.

The probability distribution of the mean reactant mass fraction at different positions through the mean flame location agrees with the wrinkled-laminar-flame model of Lieuw, Bray & Moss (1981) consisting of two delta functions representing pure reactant and pure product joined by a region of low probability proportional to the inverse of the reactant mass-fraction gradient.

Hill, *Leonard*, *Rogers* and *Rutland* modified Rogallo's code to begin a study of a simple diffusion chemical reaction in incompressible homogeneous turbulence. The objective was to determine the influence of vorticity and strain rate on the structure and propagation of the reaction zone (flame).

They considered a simple irreversible chemical reaction $A+B+C \rightarrow P+C$ occurring in isotropic turbulence with the reaction rate proportional to the product of the local concentrations of non-premixed reactants A and B through a constant reaction-rate coefficient. P denotes the product, and C is an inert diluent. The mass fractions of the four species were computed along with the hydrodynamic field on a $64 \times 64 \times 64$ grid. The initial chemical composition consisted of alternating slabs of pure reactants A+C and B+C. The Schmidt number was unity for all species. Two runs with different reaction rates resulted in Damkohler numbers based in the initial mean reactant concentration, turbulence intensity, and integral velocity scale of 2 and 10.

The instantaneous reaction front appeared more convoluted in this study than in the premixed-flame calculation described previously, presumably owing at least in part to the higher Reynolds number of this simulation. The structure of surfaces of constant reactant concentration appear to be well correlated with the local velocity field. The simulation indicates that the dissipation microscale of the concentration fields is not greatly affected by the Damkohler number, suggesting that the diffusive effects can be treated reasonably well by correctly modelling an equivalent non-reacting flow.

These observations may be helpful in modelling reacting flows. This study is leading to an examination of the effects of mean strain and shear on the flames, variation of the reactant Schmidt numbers, sensitivity to initial conditions, and non-stoichiometric mixtures.

7. Concluding remarks

The objectives of this summer programme lasting 4 weeks were ambitious: to define in computational terms the tests that were necessary to investigate certain models and concepts and then to write the software to retrieve, analyse and display vast quantities of data from the database of the computed flow field. (In some cases new flow fields had even to be computed.) This required commitment to the programme of much of the capacity of the main frame computers of NASA Ames (CRAY II and CRAY XMP), as well as the advanced workstations. It also required an intensive supporting effort by the researchers at the Center for Turbulence Research.

The programme showed that an intensive and well-prepared collaborative effort in examining databases of turbulent velocity fields does lead to a critical evaluation and better understanding of current models and concepts. But the collaboration and discussion also led to some new ideas and some new modelling techniques which can certainly be used for practical fluid flow problems. As important was the demonstration that with the same data being used for quite different interpretations and model testing, it was possible to see what aspects of turbulence can be best understood by different theoretical, computational and experimental techniques, e.g. by dynamical system theory and nonlinear stability theory for the sublayer; by conditional sampling for coherent structure; by statistical methods for interactions between different scales, or interactions over distances.

This summer programme certainly helped us to see how these methods can be complementary; for example, statistical computations or measurements are useful for the synthesis of conditional eddies, or for deriving the structure of suitable low-order dynamical systems.

Finally, perhaps it was because the summer programme was such an intensive collaborative working effort by those attending that it has been successful in promoting collaboration between many of the participants afterwards.

My thanks to the organizers of this summer programme for being invited to participate in such an interesting project. I am grateful to P. Moin and P. Bradshaw, in particular, for helping with this report, which is based on a draft produced by the organizers of the groups.

REFERENCES

Ashurst, W. T., Kerstein, A. R., Kerr, R. M. & Gibson, C. H. 1987 Alignment of vorticity and scalar gradient with strain rate in simulated Navier–Stokes Turbulence. *Phys. Fluids*. **30**, 2343–2353.

Aubry, N., Holmes, P., Lumley, J. L. & Stone, E. 1987 Models for coherent structures in the wall layer. In *Proc. Advances in Turbulence* (ed. G. Comte-Bellot & J. Mathieu), pp. 346–356. Springer.

Bardina, J., Ferziger, J. H. & Rogallo, R. S. 1985 Effect of rotation on isotropic turbulence: computation and modelling. *J. Fluid Mech.* **154**, 321–336.

Blackwelder, R. F. & Kaplan, R. E. 1976 On the wall structure of the turbulent boundary layer. *J. Fluid Mech.* **76**, 89–112.

Cambon, C., Jeandel, D. & Mathieu, J. 1981 Spectral modelling of homogeneous non-isotropic turbulence. *J. Fluid Mech.* **104**, 247–262.

Carruthers, D. J. & Hunt, J. C. R. 1986 Velocity fluctuations near an interface between a turbulent region and a stably stratified layer. *J. Fluid Mech.* **165**, 475–501.

Chase, D. M. 1969 Space–time correlations of velocity and pressure and the role of convection for homogeneous turbulence in the universal range. *Acustica* **22**, 303–320.

CHOLLET, J. 1982 Two-point closures as a subgrid scale modeling for large eddy simulations. *Fourth Symp. on Turbulent Shear Flows, Karlsruhe, September 1983.*

HERZOG, S. 1986 The large-scale structure in the near wall region of turbulent pipe flow. Ph.D. thesis, Cornell University.

HUNT, J. C. R. 1984 Turbulence structure in thermal convection and shear-free boundary layers. *J. Fluid Mech.* **138**, 161–184.

HUNT, J. C. R., KAIMAL, J. C. & GAYNOR, J. E. 1988 Eddy structure in the convective boundary layer – new measurements and new concepts. *Q. J. R. Met. Soc.* (In the Press.)

HUNT, J. C. R., STRETCH, D. D. & BRITTER, R. E. 1986 Length scales in stably stratified turbulent flows and their use in turbulence models. *Proc. I.M.A. Conf. on Stably Stratified flow and Dense Gas Dispersion.* Clarendon.

HUSSAIN, A. K. M. F. 1986 Coherent structures and turbulence. *J. Fluid Mech.* **173**, 303–356.

HUSSAIN, A. K. M. F. & CLARK, A. R. 1981 On the coherent structure of the axisymmetric mixing layer: a flow visualization study. *J. Fluid Mech.* **104**, 263–294.

JIMENEZ, J. 1987 Bifurcations and bursting in two-dimensional Poiseuille flow. *Phys. Fluids.* **30**, 3644–3646.

KIM, J., MOIN, P. & MOSER, R. D. 1987 Turbulence statistics in fully developed channel flow at low Reynolds number. *J. Fluid Mech.* **177**, 133–166.

KOLLMANN, W. & JANICKA, J. 1985 The probability density function of a passive scalar in turbulent shear flows. *Phys. Fluids* **25**, 1755–1982.

KRAICHNAN, R. H. 1976 Eddy viscosity in two and three dimensions. *J. Atmos. Sci.* **33**, 1521–1536.

LANDAHL, M. T. 1972 Wave mechanics of breakdown. *J. Fluid Mech.* **56**, 775–802.

LANDAHL, M. T. 1975 Coherent structures in turbulence and Prandtl mixing length theory. *Z. Flugwiss Weltraumforschung* **8**, 233–242.

LAUNDER, B. E., REECE, G. J. & RODI, W. 1975 Progress in the development of a Reynolds-stress turbulence closure. *J. Fluid Mech.* **68**, 537–566.

LEE, C. H., SQUIRES, K., BERTOGLIO, J. P. & FERZIGER, J. 1988 Study of Lagrangian characteristic times using direct numerical simulation of turbulence. *Sixth Symp. on Turbulent Shear Flows, Toulouse, France.* Springer (to be published).

LIEUW, S. K., BRAY, L. N. C. & MOSS, J. B. 1981 A flamelet model of turbulent non-premixed combustion. *Combust. Sci. Tech.* **27**, 69–73.

LESIEUR, M. 1987 *Turbulence in Fluids.* Dordrecht: Martin Nijhoff.

LUMLEY, J. L. 1965 Interpretation of time spectra in high intensity shear flows. *Phys. Fluids* **8**, 1056–1062.

MANSOUR, N. N., KIM, J. & MOIN, P. 1988 Reynolds stress and dissipation rate budgets in a turbulent channel flow. *J. Fluid Mech.* (in press).

MICHARD, M., MATHIEU, J., MOREL, R., ALCARAZ, E. & BERTOGLIO, J. P. 1987 Grid-generated turbulence exhibiting a peak in the spectrum. In *Proc. Advances in Turbulence* (ed. G. Comte-Bellot & J. Mathieu), pp. 163–169. Springer.

MOIN, P., ADRIAN, R. J. & KIM, J. 1987 Stochastic estimation of organised structures in turbulent channel flow. In *Proc. 6th Symp. on Turbulent Shear Flow, Toulouse, Paper* 16–19.

MOIN, P. & MOSER, R. D. 1988 Characteristic eddy decomposition of turbulence in a channel. *J. Fluid Mech.* (submitted).

ROGALLO, R. S. 1981 Numerical experiments in homogeneous turbulence. *NASA TM* 81315.

ROGALLO, R. S. & MOIN, P. 1984 Numerical simulation of turbulent flows. *Ann. Rev. Fluid Mech.* **16**, 99–137.

ROGERS, M. M. & MOIN, P. The structure of the vorticity field in homogeneous turbulent flows. *J. Fluid Mech.* **176**, 33–36.

ROGERS, M. M., MOIN, P. & REYNOLDS, W. C. 1986 The structure and modeling of the hydrodynamic and passive scalar fields in homogeneous turbulent shear flows. *Rep.* TF-15. Dept. Mech. Engng, Stanford University.

SCHIESTEL, R. 1987 Multiple-time-scale modelling of turbulent flows in one-point closures. *Phys. Fluids* **30**, 722–731.

SHIH, T.-H. & LUMLEY, J. L. 1985 Modeling of pressure correlation terms in Reynolds stress and scalar flux equations. Rep. FDA-85-3, Sibley School of Mech. & Aero. Eng. Cornell University, Ithaca, N.Y. (*J. Fluid Mech.* submitted).

SPALART, P. R. 1986 Numerical study of sink-flow boundary layers. *J. Fluid Mech.* **172**, 307–328.

TENNEKES, H. 1975 Eulerian and Lagrangian time microscales in isotropic turbulence. *J. Fluid Mech.* **67**, 561–567.

WEINSTOCK, J. 1981 Theory of pressure–strain-rate correlation for Reynolds-stress turbulence closure. Part 1. Off-diagonal element. *J. Fluid Mech.* **105**, 369–395.

WEINSTOCK, H. 1982 Theory of pressure–strain rate. Part 2. Diagonal element. *J. Fluid Mech.* **116**, 1–29.

WEINSTOCK, J. & BURK, S. 1985 Theoretical pressure–strain term, experimental comparison and resistance to large anisotropy. *J. Fluid Mech.* **154**, 429–443.

WILLS, J. A. B. 1971 Measurements of the wave-number/phase velocity spectrum of wall pressure beneath a turbulent boundary layer. *J. Fluid Mech.* **45**, 65–90

YOSHIZAWA, A. 1985 A statistically derived system of equations for turbulent shear flows. *Phys. Fluids* **28**, 59–63.

YEUNG, P. K. & POPE, S. B. 1987 Lagrangian velocity statistics obtained from direct numerical simulations of homogeneous turbulence. In *Proc. 6th Symp. of Turbulent Shear Flow, Toulouse,* Paper 3–7.